Matlab Assignment for 251

General Information:

- The Matlab website is full of useful information if you want to learn more Matlab Support.
- The University Software Portal has information on how to download Matlab or if preferred how to work with the online version of Matlab.
- The solutions for this assignment should consist of a document (for example a word file) where you include all the images you generated, plus a copy of the code you wrote on Matlab (or Mathematica/Maple if you chose to use these instead).
- Depending on the version you are using of Matlab, there might be slightly changes in the wording of some of the codes which appear on this assignment, but you can find what the differences are by using Matlab Support Matlab Support.

Part 1

Parametrizing Curves:

Viviani's curve is defined as the intersection of the sphere $x^2 + y^2 + z^2 = 4$ with the cylinder $(x-1)^2 + y^2 = 1$. Observe that

$$\begin{cases} x = 1 + \cos t \\ y = \sin t \\ z = \pm 2 \sin\left(\frac{t}{2}\right) \end{cases}$$

gives a parametrization of this curve [this uses the identity $\sin\left(\frac{t}{2}\right) = \pm\sqrt{\frac{1-\cos t}{2}}$].

To plot Viviani's curve use the following code

```
>> syms t; x=1 +cos(t); y=sin(t), z=2*sin(t/2);
>> fplot3(x,y,z, [0 pi])
```

Exercise 1. Save the image you get as a pdf or jpeg file.

The figure you obtained should not be terribly illuminating. We can get rid of the grid that by default surrounds all three-dimensional graphics in Matlab. Also, we can dispense with the labels and tick marks on the axes. It is also helpful to show the curve inside the sphere and the cylinder that are used to define it, and show the arcs obtained by intersecting with the coordinate planes, as well as the projected circle on the xy plane.

```
>> hold on
>> fplot3(sym(0), 2*cos(t/2), 2*sin(t/2), [0 pi])
>> fplot3(2*cos(t/2), sym(0), 2*sin(t/2), [0 pi])
>> fplot3(2*cos(t/2), 2*sin(t/2), sym(0), [0 pi])
>> fplot3(1+cos(t), sin(t), sym(0), [0 pi])
>> fplot3(sym(0), sym(0), t, [0 2])
>> fplot3(sym(0), t, sym(0), [0 2])
>> fplot3(t, sym(0), sym(0), [0 2])
>> title(' '); xlabel(' '); ylabel(' '); zlabel(' ');
>> grid off; axis off;
view ([10,3,1])
```

Exercise 2. Save the image you get as a pdf or jpeg file.

Exercise 3. Identify which fplot3 commands correspond to

- **The x-axis, y-axis, z-axis**
- **Circle arc on the xy plane, xz plane, yz plane**
- **Circle $(x-1)^2 + y^2 = 1$ on the xy plane**
- **You can enter comments on Matlab directly by using the % symbol.**

Exercise 4. Now plot the curve which is obtained as the intersection of the paraboloid $z = x^2 + y^2$ with the cylinder $(x-1)^2 + y^2 = 1$. Use the interval [0 2*pi] instead of [0 pi]. Hint: notice that this is the same cylinder as before so x and y are given by the same functions of t. Only z changes.

Newton's Laws:

One can also use Matlab to solve differential equations, which might be handy if you take MATH-244 or a similar course! For example, for a charged particle with charge q in the presence of a magnetic field $\mathbf{B}$, Newton's Law reads

$$m\frac{d^2\mathbf{r}}{dt^2} = \frac{q}{c}\frac{d\mathbf{r}}{dt} \times \mathbf{B} = -\frac{q}{c}\mathbf{B} \times \frac{d\mathbf{r}}{dt}$$

where c is the speed of light in vacuum and $\mathbf{r}(t) = (x(t), y(t), z(t)) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$

To illustrate how matlab works, suppose the initial velocity is

$$\mathbf{v}(0) = (v_1, v_2, v_3) = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$$

and there is a constant magnetic field

$$\mathbf{B} = (0, 0, B) = B\mathbf{k}$$

(so here B is constant!).

Exercise 5. Write down the differential equation that $z(t)$ satisfies and show that the motion has constant speed in the z direction.

Write

```
>> syms q c m t v1 v2 v3 B x(t) y(t) z(t)
>> rhs = (-q*B/ (c*m) ) * cross ( [0,0,1] , [diff(x), diff(y), diff(z)])
>> Dx=diff(x,t)
>> Dy=diff(y,t)
>> Dz=diff(z,t)
>> sol = dsolve([diff(x,2), diff(y,2), diff(z,2)] == rhs, [x(0), y(0), z(0)] == [0,0,0], [Dx(0), Dy(0), Dz(0)] ==[v1,v2,v3]);
```

Exercise 6. Use

```
>> sol.z
```

to verify the previous exercise (write what you get on matlab)

Exercise 7. Take $v_1 = 0$,
$v_2 = B = q = c = m = 1$ and write the command

```
>> assume(t, 'real'); assume(v3, 'real'); simplesol=simplify(subs([sol.x,sol.y,sol.z],[c,m,q,B,v1,v2],[1,1,1,1,0,1]))
```

What solution do you get?

Exercise 8. Your solution might be worrisome since $x(t)$ involves complex numbers! Using that (Euler's formula)

$$e^{it} = \cos t + i \sin t$$

rewrite $x(t)$ using only trigonometric functions.

Example 9. Can you identify the curve that you get? You may plot it if it helps!

Part 2

Graphing Functions:

fsurf and fmesh are the basic MATLAB commands for graphing a function of two variables. For example, suppose we want to graph the function

$$f(x, y) = y\left(1 - \frac{1}{(x^2 + y^2)}\right)$$

Exercise 10. What is the domain of this function?

To plot it write

```
>> flowfunction= @(x,y) y*(1-1/(x^2+y^2));
>> figure; fsurf(flowfunction, [-2 2 -2 2])
```

You will receive a warning because the function is not defined everywhere on the xy plane but that is ok.

Exercise 11. Save the image you get as a pdf or jpeg file.

To graph level curves we can use:

```
>> list1= -2:0.5:2; figure; fcontour(flowfunction, [-2 2 -2 2], 'LevelList', list1);
```

Exercise 12. Save the image you get as a pdf or jpeg file.

To shade the regions between the level curves selected use

```
>> figure; fcontour(flowfunction, [-2 2 -2 2], 'LevelList', list1, 'Fill', 'on')
```

Exercise 13. Save the image you get as a pdf or jpeg file.

Exercise 14. Plot the function $f(x, y) = (x^2 + y^2)^2 - x^2 + y^2$, and define list1=[-0.2, -0.1], list2=[0.1, 0.2, 1, 4, 10], list3=[0] Compare the level curves of f for these different values. Save the images you get. Use the grid [-1 1 -1 1]

```
>> flowfunction= @(x,y) (x^2+y^2)^2 -x^2+y^2;
>> figure; fsurf(flowfunction, [-1 1 -1 1])
```

```
>> list1= [-0.2, -0.1]; figure; fcontour(flowfunction, [-1 1 -1 1], 'LevelList', list1);
```

```
>> list2= [0.1, 0.2, 1, 4, 10]; figure; fcontour(flowfunction, [-1 1 -1 1], 'LevelList', list2);
```

```
>> list3= [0]; figure; fcontour(flowfunction, [-1 1 -1 1], 'LevelList', list3);
```

Partial Derivatives and the Gradient

In Matlab, partial derivatives are computed with the diff command:

```
>> syms x y; diff(x^3+sin(x*y),x)
```

Before we defined the "flowfunction". If you want to compute its gradient you can use the command Jacobian.

```
>> gradflowfunction =jacobian(flowfunction(x,y))
```

If you want to plot the previous gradient, one should use the command "quiver". Display

```
>> [X,Y]= meshgrid(-2:0.2:2, -2:0.2:2);
>> U= (2.*X.*Y)./((X.^2+Y.^2).^2);
>> V= (2.*Y.^2). /((X.^2+Y.^2).^2)-1./(X.^2+Y.^2)+1;
>> L=sqrt(U.^2+V.^2);
>> figure; gradientfield= quiver(X,Y, U./L, V./L, 0.5)
```

Exercise 15. Show the image that you get.

Part 3

Surfaces and Vector Fields:

```
>> syms x y z; h=z^2 -x^2-y^2;
>> fimplicit3(h-1, [-1.1 1.1 -1.1 1.1 -2 2]); axis equal
```

Exercise 16. Save the image you get as a pdf or jpeg file.

With the command quiver3 we can visualize gradient vectors. Write

```
>> [X,Y]=meshgrid(-1:0.5:1., -1:0.5:1.);
>> Z=sqrt(1+X.^2+Y.^2);
>> [U,V,W]=surfnorm(X,Y,Z);
>> hold on; quiver3(X,Y,Z,U,V,W,2)
>> Z1= -sqrt(1+X.^2+Y.^2);
>> [U,V,W]= surfnorm(X,Y,Z1);
>> quiver3(X,Y,Z1,U,V,W,2)
```

Exercise 17. Save the image you get as a pdf or jpeg file.

Exercise 18. Plot the ellipsoid $f(x,y,z) = x^2+2y^2+3z^2 = 1$ together with the tangent plane at the point $(1,0,0)$. You need to use the command hold on; **in order to have both images appear simultaneously.**

Parametrizing Surfaces:

A parametrization of the sphere $x^2 + y^2 + z^2 = 1$ in spherical coordinates θ, ϕ:

$$\begin{cases} x = \cos u \sin v \\ y = \sin u \sin v \\ z = \cos v \end{cases}$$

Here u is playing the role of θ while v is playing the role of ϕ.

To plot the sphere use

```
>> syms u v real; sphere= [cos(u)*sin(v), sin(u)*sin(v), cos(v)];
>> fsurf(sphere(1), sphere(2), sphere(3), [0 2*pi 0 pi])
>> view([1,1,1]); title(' '); xlabel(' '); ylabel(' '); zlabel(' ');
>> grid off; axis equal; axis off;
```

Exercise 19. Save the image you get as a pdf or jpeg file.

Exercise 20. Parametrize the monkey saddle $z = x^3 - 3xy^2$ by using $u = x$, $v = y$ as the parameters. Notice that here it makes more sense to use both negative and positive values of u, v

Exercise 21. Save the image you get as a pdf or jpeg file.